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The bullwhip effect on product orders and inventory: a perspective of demand forecasting techniques

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Demand forecasting is one of the key causes of the bullwhip effect on product orders. Although this aspect of order oscillation is not ignored, the current study focuses on another critical aspect of oscillation: the bullwhip effect on inventory, i.e. the net inventory variance amplification. In particular, this paper studies a two-level supply chain in which the demand is price sensitive, while the price follows a first-order autoregressive pricing process. We derive the analytical expressions of the bullwhip effect on product orders and inventory using minimum mean-squared error, moving average and exponential smoothing forecasting techniques. We also propose the conditions under which the three forecasting techniques would be chosen by the retailer to minimise the sum of the bullwhip effect on product orders and inventory under different weightings. These observations are used to develop managerial insights regarding choosing an appropriate forecasting technique after considering certain distinct characteristics of the product.

Keywords: bullwhip effect on product orders; bullwhip effect on inventory; oscillation; forecasting technique; supply chain management

1. Introduction

Accurate demand forecasting is crucial to inventory planning and control because estimating the lead time demand helps set an appropriate level of stock anticipation (Zhang and Zhao 2010). However, it has been recognised that demand forecasting is one of the key causes of the bullwhip effect (Lee *et al.* 1997a). The bullwhip effect is a phenomenon of exaggeration because as ordering information percolates upstream, a demand fluctuation downstream leads to larger fluctuations in upstream orders and inventories (Lee *et al.* 1997a, 1997b, Gilbert 2005).

The connection between forecasting and the bullwhip effect is surmised from the fact that the mean demand is estimated by the supply chain downstream based on forecasting. However, both upstream and downstream businesses have a fundamental interest in forecasting. Upstream businesses desire that the forecasting technique chosen by downstream businesses would restrain the order variance amplification, i.e. the bullwhip effect on product orders. Additionally, downstream businesses wish that the chosen forecasting technique would restrain the inventory variance amplification.[1] In particular, the bullwhip effect on product orders can lead to misguided capacity plans, missed production schedules and inactive transportation from upstream businesses (Lee *et al.* 1997b). In other words, the bullwhip effect on product orders mainly contributes to upstream costs, while inventory oscillations motivate high levels of safety stock and make downstream large inventory costs unavoidable (Hoberg *et al.* 2007). Thus, the bullwhip effect on inventory impacts downstream costs. It is interesting to note that the problem described above may lead to non-cooperative behaviour. It is counterintuitive that a downstream stage should be concerned with upstream costs. Likewise, the converse is also counterintuitive. This is the key tradeoff faced by a single-stage member of a supply chain. The main objective of this paper is to analyse the effect of different forecasting techniques on the bullwhip effect on product orders, the bullwhip effect on inventory and the sum of the two oscillations under different weightings.

Three basic types of forecasting technique are focused on by recent researchers and practitioners: the minimum mean-squared error (MMSE) technique, the moving average (MA) technique and the exponential smoothing (ES) technique. MMSE is an optimal forecasting procedure that minimises the mean-squared forecasting error. In the

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area of forecasting, an 'optimal' forecasting model traditionally implies that the forecasting model has minimal mean-squared forecasting errors (Alwan *et al*. 2003). In general, the MA and ES forecasting techniques do not share this optimal property for a time series process (Zhang 2004). However, MA and ES are the most commonly used forecasting techniques in practice as a result of their ease of use, flexibility and robustness (Ryan 1997, Chen *et al*. 2000a, 2000b). This paper examines the differences in the bullwhip effect on product orders and inventory measures when the three forecasting techniques are used to forecast lead time demand.

In this paper, we consider a two-level supply chain with a manufacturer and a retailer in which the demand faced by the retailer is price sensitive. The price follows dynamics with a first-order autoregressive (AR (1)) pricing process, and different product characteristics are considered, i.e. a different market demand scale, price sensitivity coefficient, price correlation coefficient and lead time. From this, we derive the analytical expressions of the bullwhip effect on product orders and inventory with MMSE, MA and ES techniques and deduce the conditions under which the retailer chooses a forecasting technique to minimise the weighted sum of the two variance amplifications.

This paper is organised in the following manner. Section 2 reviews the literature. Section 3 establishes a price-sensitive demand function in which the price follows an AR (1) pricing process. Section 4 introduces the ordering policy model. Section 5 derives the analytical expressions of the bullwhip effect on product orders with MMSE, MA and ES techniques and compares the order oscillations for the three forecasting techniques. Section 6 investigates the bullwhip effect on inventory at the retailer and compares the results for the three forecasting techniques. Section 7 introduces a new performance measure, which is the sum of the bullwhip effect on product orders and the bullwhip effect on inventory under different weightings, and derives the conditions for choosing the best forecasting technique to minimise the sum of the two oscillations regarding certain product characteristics. Section 7 also provides a numerical study to explain the conditions. Section 8 presents the conclusions and suggests some follow-up research directions.

2. Literature review

Our research builds on two lines of literature: the literature on the bullwhip effect on product orders and the literature on the bullwhip effect on inventory.

2.1 *Bullwhip effect on product orders*

Over the past few decades, the bullwhip effect on product orders has become a popular topic for researchers and practitioners. Early studies attempted to demonstrate the existence of the bullwhip effect on product orders and identify the causes of such an effect (Forrester 1958, 1961, Sterman 1989). Currently, most papers focus on quantifying and searching for remedies for the bullwhip effect on product orders. Lee *et al*. (1997a, 1997b) provided a formal definition of the bullwhip effect on product orders and systematically analysed four main causes: demand signal processing, shortage games, order batching and price adjustment. In addition, they proposed the counter-measures: avoid multiple demand forecast updates, break order batches, stabilise prices and eliminate gaming in shortage. Chen *et al*. (2000a, 2000b) made a great contribution by recognising the role of demand forecasting as a filter for the bullwhip effect on product orders. Chen *et al*. (2000a) quantified the bullwhip effect on product orders for a two-level supply chain in which the retailer used the MA forecasting technique, and extended these results to multiple-stage supply chains with and without centralised demand information. In a sequel, Chen *et al*. (2000b) demonstrated that the use of ES technology by the retailer can also create the bullwhip effect on product orders. These results were contrasted with the increase in variability caused by the use of MA technology. However, although MA and ES are the most commonly used forecasting techniques in practice, they are not optimal forecasting techniques. Alwan *et al*. (2003) studied the bullwhip effect on product orders when MMSE forecasting was employed instead of other commonly used simplistic forecasting schemes. They found that it was possible to reduce or even eliminate this effect by using an MMSE-optimal forecasting scheme. Similar work has also been conducted by Zhang (2004), Duc *et al*. (2010) and Sodhi and Tang (2011). These studies have examined the bullwhip effect on product orders by assuming that the demand follows an autoregressive process and the retailer employs an order-up-to inventory policy. Subsequent papers extended their work by allowing for more general time-series demand processes (Aviv 2003, Gilbert 2005, Disney *et al*. 2006, Hsiao and Shieh 2006, Dhahri and Chabchoub 2007, Duc *et al.* 2008) or different inventory policies (Aviv 2003, Dejonckheere *et al.* 2003, 2004, Wadhwa *et al.* 2009, Wang *et al.* 2010).

2.2 *Bullwhip effect on inventory*

The bullwhip effect on inventory may lead to large inventory costs for downstream businesses and generate depressed customer service levels. Several authors have used different methods to study inventory oscillation. Disney and Towill (2003) and Disney *et al.* (2004) derived an analytical expression for the variance of the inventory level, and combined this expression with the expression of the bullwhip effect on product orders to determine suitable ordering system designs through discrete control theory and z-transform techniques. Disney *et al.* (2006) extended this model to study a generalised order-up-to policy in terms of orders and inventory variance and the customer service levels it generates. They quantified the bullwhip effect on product orders and the bullwhip effect on inventory for independent and identically distributed (IID), auto-regressive, moving-average and auto-regressive moving-average demand processes. Similar work has been also conducted by Gaalman and Disney (2006), Hosoda and Disney (2006), Disney *et al.* (2007), Hoberg *et al.* (2007) and Boute *et al.* (2008). The above authors studied the bullwhip effect on inventory with linear control theory. Other methods have also been used in other studies. Kim and Springer (2008) studied the two volatilities through a system dynamics model and derived the conditions for generating oscillation. In a sequel, Springer and Kim (2010) also used a system dynamics approach to study supply chain volatility. Coppini *et al.* (2010) used Matlab 7.0 to simulate the four-stage beer game supply chain model. They showed that inventory oscillations provided more information on supply chain performance than the bullwhip effect on product orders measure. The study presented in this paper uses the statistical method and extends the work of Chen *et al.* (2000a, 2000b) and Lee *et al.* (2000) by including different demand processes and different forecasting techniques.

The contributions of this paper are threefold. First, in the previous research, the demand follows an autoregressive process, and the demand correlation parameter on the bullwhip effect on product orders was discussed, as in Ryan (1997), Lee *et al.* (1997a), Chen *et al.* (2000a, 2000b), Lee *et al.* (2000), Zhang (2004, 2005) and Sodhi and Tang (2011). However, the managerial insights of this parameter are difficult to explain in practice. Our research will consider a price-sensitive demand function in which the price follows an AR (1) pricing process. This will allow us to focus on a different perspective to explain the impact of demand process characteristics, including the market demand scale, the price sensitivity coefficient and the price correlation parameter on the bullwhip effect on product orders. Second, we derive the analytical expressions of the bullwhip effect on inventory with MMSE, MA and ES techniques using a statistical method. The introduction of the bullwhip effect on inventory has the potential to provide some insights on the consequences that inventory oscillations have on downstream management costs, because previous research on the product order oscillations focuses on the upstream management cost (Lee *et al.* 2000, Kim and Ryan 2003, Zhang 2004). Third, we introduce a new performance measure of the bullwhip effect based on the weighted sum of the order variance and inventory variance amplifications, and derive the conditions for choosing among the three forecasting techniques the one that best minimises the bullwhip effect. Therefore, this paper analyses the retailer's forecasting techniques from the perspective of an entire supply chain, rather than only from the perspective of the upstream stage. For example, research on the causes and remedies for the bullwhip effect on product orders is more pertinent for the upstream side because the bullwhip effect on product orders mainly contributes to upstream costs.

3. Demand model

Consider a simple two-level supply chain with a manufacturer and a retailer. External demand for a single product occurs at the retailer, where the demand faced by the retailer is price sensitive. Let d_t and p_t be the customer demand and market price in period t, respectively. We obtain the basic linear demand function model as follows:

$$d_t = D(p_t, \varepsilon_{1,t}) = a - bp_t + \varepsilon_{1,t}, \quad b > 0, \tag{1}$$

where a refers to the market demand scale, b is the price sensitivity coefficient and $\varepsilon_{1,t}$ is an IID normally distributed error term across time with mean zero and variance σ_1^2. We further assume that $\varepsilon_{1,t}$ has no relation with the market price. Therefore, the covariance structure between the error term and the market price is the following: $Cov(p_t, \varepsilon_{1,t'}) = 0$ for any t or t' (including the situation of $t = t'$ and the situation of $t \neq t'$).

We consider a market setting in which the retailer sells on a perfectly competitive market and exerts no control over the market clearing price. We incorporate price dynamics in our demand model, and the market price evolution is determined by the overall market demand and supply. Let market price p_t in Equation (1) be an AR (1) pricing process that describes price dynamics:[2]

$$p_t = \mu + \rho p_{t-1} + \varepsilon_{2,t}, \quad -1 < \rho < 1, \tag{2}$$

where μ is a nonnegative constant that determines the mean of the price, ρ is the price correlation coefficient, and $\varepsilon_{2,t}$ is an IID normally distributed error term with mean zero and variance σ_2^2. The condition of $-1 < \rho < 1$ ensures that the AR (1) pricing process is stationary.[3] When the coefficient ρ is negative, the process tends to exhibit period-to-period oscillatory. On the other hand, when the coefficient ρ is positive, the process is reflected by a wandering or meandering sequence of observations. In particular, if the coefficient ρ has a large positive value, neighbouring values in the process are similar and the process exhibits marked trends. Also, when the value of ρ is zero, we have an IID process with mean μ and variance σ_2^2. Therefore, by taking different values of the coefficient ρ, one can represent a wide variety of pricing process behaviours. See Box and Jenkins (1994) for a more detailed discussion of this coefficient. Similar to Zhang and Burke (2011), we assume that the error term $\varepsilon_{2,t}$ and the market price have a covariance structure that $Cov(p_t, \varepsilon_{2,t'}) = 0$ if $t < t'$. It can easily be shown from Equation (2) that $E(p_t) = \mu/(1-\rho)$ and $\sigma_p^2 = Var(p_t) = \sigma_2^2/(1-\rho^2)$ and it can be shown from Equation (1) that $E(d_t) = a - b\mu/(1-\rho)$ and $\sigma_d^2 = Var(d_t) = \sigma_1^2 + b^2\sigma_2^2/(1-\rho^2)$. Note that we have already assumed that the covariance $Cov(p_t, \varepsilon_{1,t'}) = Cov(\mu + \rho p_{t-1} + \varepsilon_{2,t}, \varepsilon_{1,t'}) = Cov(\varepsilon_{2,t}, \varepsilon_{1,t'}) = 0$ for any t or t'. In other words, the error terms are independent across time and are not correlated contemporaneously.

We can derive from Equations (1) and (2) that $d_t = \mu' + \lambda d_{t-1} + \varepsilon_t$, where $\mu' = a(1-\rho) - b\mu$, $\lambda = \rho$ and $\varepsilon_t = \varepsilon_{1,t} - \rho\varepsilon_{1,t-1} - b\varepsilon_{2,t}$. The model that describes the demand in Equation (1) and the price dynamics in Equation (2) can be reduced to an autoregressive demand process. It should be noted that ε_t is the function of the two kinds of error terms, $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$. Thus, the reduced demand model is not an AR (1) process. However, in the previous research, an AR (1) demand process was adopted by most researchers (Lee *et al.* 1997a, Chen *et al.* 2000a, 2000b, Lee *et al.* 2000, Zhang 2004, 2005). They investigated the bullwhip effect on product orders as a function of a demand process characteristic, namely, the demand correlation parameter λ. However, it is difficult to explain its managerial insights in practice. Our research considers a price-sensitive demand function in which the price is an AR (1) pricing process. Therefore, we focus on a different perspective to explain the impact of demand process characteristics, including the market demand scale a, the price sensitivity coefficient b and the price correlation coefficient ρ, on the bullwhip effect on product orders and the bullwhip effect on inventory. This analysis provides us with more managerial insights in our research.

4. Ordering process

We adopt the following timing of events during the replenishment period. The retailer observes consumer demand d_{t-1} at the end of period $t-1$ and places an order of quantity q_t to the manufacturer at the beginning of period t according to its current inventory level. After the lead time L, the retailer receives the product from the manufacturer at the beginning of period $t+L$. We may then describe the net inventory level of the retailer as follows:

$$I_t = I_{t-1} + q_{t-L} - d_{t-1}, \tag{3}$$

where I_t is the net inventory at the beginning of period t. The net inventory is the on-hand inventory, which is the stock you have minus any backorders (Hosoda and Disney 2006). A negative value of net inventory indicates a backorder condition on customer demand.

We can also rewrite Equation (3) as the following:

$$q_t = I_{t+L} - I_{t+L-1} + d_{t+L-1}. \tag{4}$$

Vassian (1955) gives an order policy that provides the minimum variance of net inventory level over time as follows:

$$q_t = \hat{D}_t^L - \sum_{i=1}^{L-1} q_{t-i} - I_t, \tag{5}$$

where $\hat{D}_t^L$ is an estimate of the mean lead time demand and $\sum_{i=1}^{L-1} q_{t-i}$ is the total orders that are already placed but are not yet received. See Vassian (1955) for a more detailed discussion of Equation (5).

Substituting Equation (4) into the two sides of Equation (5), we obtain the following:

$$\begin{aligned} I_{t+L} - I_{t+L-1} + d_{t+L-1} &= \hat{D}_t^L - \sum_{i=1}^{L-1}(I_{t+L-i} - I_{t+L-1-i} + d_{t+L-1-i}) - I_t \\ &= \hat{D}_t^L - \left(I_{t+L-1} - I_t + \sum_{i=0}^{L-2} d_{t+i}\right) - I_t. \end{aligned} \tag{6}$$

After some simplification of Equation (6), we find that:

$$I_{t+L} = \hat{D}_t^L - \sum_{i=0}^{L-1} d_{t+i} = \hat{D}_t^L - D_t^L. \tag{7}$$

Consequently,

$$I_t = \hat{D}_{t-L}^L - D_{t-L}^L. \tag{8}$$

We notice from Equation (8) that the net inventory in period t is the same as the lead time demand forecasting error made in period $t-L$. Therefore, we can compute variance of the lead time demand forecasting error instead of computing the net inventory variance (Gaalman and Disney 2006, Hosoda and Disney 2006). Hence, we obtain $Var(I_t) = Var(\hat{D}_{t-L}^L - D_{t-L}^L)$.

4.1 *Order-up-to policy*

Let q_t represent the order quantity at the beginning of period t; let y_t be the order-up-to level used in period t; the ordering decision in an order-up-to system is thus:

$$q_t = y_t - (y_{t-1} - d_{t-1}). \tag{9}$$

In other words, the order quantity q_t is the order-up-to level used in period t minus the inventory position at the end of period $t-1$. Notice from Equation (9) that the product order quantity q_t may be negative. If this is the case, similar to Ryan (1997) and Chen *et al*. (2000a, 2000b), we assume that this excess inventory is returned without cost. See Chen *et al*. (2000b) for a more detailed discussion of these contents.

In the literature on supply chain models, order-up-to policy is one of the most studied policies, which was previously reported by Lee *et al*. (1997a) and then adopted and modified by Chen *et al*. (2000a, 2000b) and Dejonckheere *et al*. (2004). This inventory policy can also be seen in Lee *et al*. (2000), Zhang (2004, 2005), Hosoda and Disney (2006), Zhang and Zhao (2010) and Sodhi and Tang (2011). In this research, we assume that the retailer adopts the order-up-to inventory policy.

The order-up-to level is updated every period according to the following:

$$y_t = \hat{D}_t^L + z\hat{\sigma}_t^L, \tag{10}$$

where $\hat{D}_t^L$ is an estimate of the mean lead time demand, z is a constant that has been set to meet a desired service level and is often referred to as the safety factor (Chen *et al*. 2000b) and $\hat{\sigma}_t^L = \sqrt{Var(D_t^L - \hat{D}_t^L)}$ is an estimate of the standard deviation of the L period forecasting error.

Note that the order-up-to level in Equation (10) consists of an anticipation stock kept to meet the expected lead time demand and a safety stock for hedging against unexpected demand.

Hosoda and Disney (2006) verified that the ordering policies developed by Vassian (1955) (as shown in Equation (5)) are the same as the order-up-to policy (as shown in Equation (9)). However, Equation (5) is often used to generate the block diagram with control theory (Vassian 1955, Hosoda and Disney 2006). We use Equation (9) in our paper because of its simplicity. When the demand is normally distributed, Ryan (1997) and Gaalman and Disney (2006) showed that the order-up-to policy is optimal. Most papers studied the bullwhip effect on product orders by employing the order-up-to policy, for example Lee *et al*. (1997a), Chen *et al*. (2000a, 2000b), Lee *et al*. (2000) and Hosoda and Disney (2006). Because the errors $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are both IID normally distributed and are not correlated contemporaneously, we know that the price p_t and the demand d_t are also both normally distributed. This research will use the order-up-to policy to study the bullwhip effect on product orders and inventory.

Substituting Equation (10) into Equation (9), the order quantity q_t can be rewritten as the following:

$$q_t = \hat{D}_t^L - \hat{D}_{t-1}^L + d_{t-1} + z(\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L). \tag{11}$$

Because the manufacturer wants the forecasting technique that the retailer has chosen to restrain the product order variance amplification, we will first analyse the bullwhip effect on product orders with MMSE, MA and ES techniques from the perspective of the manufacturer.

5. Analytical properties of the bullwhip effect on product orders using the three forecasting techniques

The bullwhip effect on product orders is computed as the ratio of the order variance of the retailer to the demand variance of the customer (Lee *et al.* 1997a, Chen *et al.* 2000a, 2000b, Zhang 2004). If the ratio is larger than one, then the bullwhip effect on product orders is present. This information distortion means potential costs for the manufacturer, such as misguided capacity plans, missed production schedules and inactive transportation (Lee *et al.* 1997b). Therefore, the manufacturer wants the chosen forecasting technique to restrain the bullwhip effect on product orders. In this section we derive analytical expressions of the bullwhip effect on product orders with MMSE, MA and ES forecasting techniques and compare the order oscillations for the three forecasting techniques.

5.1 *The bullwhip effect on product orders using the MMSE forecasting technique*

Using the MMSE technique, it has been shown that the MMSE forecast of period $t+i$ $(i = 0, 1, 2, \ldots)$ is the conditional expectation of d_{t+i} given previous observations $d_{t-1}, d_{t-2}, \ldots$; see Box and Jenkins (1994). Let $\hat{d}_{t+i}$ be the demand forecast of period $t+i$ $(i = 0, 1, 2, \ldots)$ made at the end of period $t-1$. In the case of the AR (1) demand process, it has been shown that the MMSE forecast of $\hat{d}_{t+i}$ is given by $E(d_{t+i}|d_{t-1})$; see, e.g., Lee *et al.* (2000), Alwan *et al.* (2003), Zhang (2004), Agrawal *et al.* (2009) and Sodhi and Tang (2011). However, this paper considers a price-sensitive demand function in which the price follows an AR (1) process. Let $\hat{p}_{t+i}$ be the market price forecast of period $t+i$ made at the end of period $t-1$. Similarly, in the case of the AR (1) pricing process, this implies that $\hat{p}_{t+i}$ can be given as the future price conditioned on the actual price observed up to period $t-1$, i.e., $E(p_{t+i}|p_{t-1})$. Thus, we can derive the demand forecast of period $t+i$ that $\hat{d}_{t+i} = a - b\hat{p}_{t+i}$.[4]

By recursively applying Equation (2), it is easy to show that the following is true:

$$\begin{aligned} p_{t+i} &= \mu + \rho p_{t+i-1} + \varepsilon_{2,t+i} = (1+\rho)\mu + \rho^2 p_{t+i-2} + (\rho\varepsilon_{2,t+i-1} + \varepsilon_{2,t+i}) \\ &= \cdots = \frac{1-\rho^{i+1}}{1-\rho}\mu + \rho^{i+1}p_{t-1} + \sum_{j=o}^{i} \rho^{i-j}\varepsilon_{2,t+j}. \end{aligned} \tag{12}$$

Thus,

$$\hat{p}_{t+i} = E(p_{t+i}|\,p_{t-1}) = \frac{1-\rho^{i+1}}{1-\rho}\mu + \rho^{i+1}p_{t-1}. \tag{13}$$

Thus,

$$\hat{d}_{t+i} = a - b\hat{p}_{t+i} = a - b\left(\frac{1-\rho^{i+1}}{1-\rho}\mu + \rho^{i+1}p_{t-1}\right). \tag{14}$$

Thus,

$$\hat{D}_t^L = \sum_{i=0}^{L-1} \hat{d}_{t+i} = aL - \frac{b\mu}{1-\rho}\left(L - \frac{\rho(1-\rho^L)}{1-\rho}\right) - \frac{b\rho(1-\rho^L)}{1-\rho}p_{t-1}. \tag{15}$$

Lemma 1: *When the MMSE technique is used to forecast the lead time demand, the variance of forecasting error for the lead time demand remains constant over time and is given by the following*:

$$(\hat{\sigma}_t^L)^2 = L\sigma_1^2 + \frac{b^2}{(1-\rho)^2}\left(L + \frac{\rho(1-\rho^L)(\rho^{L+1}-\rho-2)}{1-\rho^2}\right)\sigma_2^2. \tag{16}$$

Proof: The variance of the forecasting error for the lead time demand is the following:

$$\begin{aligned}
\left(\hat{\sigma}_t^L\right)^2 &= Var\left(D_t^L - \hat{D}_t^L\right) = Var\left(\sum_{i=0}^{L-1}\left(d_{t+i} - \hat{d}_{t+i}\right)\right) = Var\left(\sum_{i=0}^{L-1}\left(-b\sum_{j=0}^{i}\rho^{i-j}\varepsilon_{2,t+j} + [\varepsilon_{1,t+i}]\right)\right)\\
&= Var\left(\sum_{i=0}^{L-1}\varepsilon_{1,t+i}\right) + b^2 Var\left(\sum_{i=0}^{L-1}\sum_{j=0}^{i}\rho^{i-j}\varepsilon_{2,t+j}\right) = L\sigma_1^2 + b^2 Var\left(\sum_{i=0}^{L-1}\varepsilon_{2,t+i}\sum_{j=0}^{L-1-i}\rho^j\right)\\
&= L\sigma_1^2 + \left(b^2/(1-\rho)^2\right)Var\left(\sum_{i=0}^{L-1}(1-\rho^{L-i})\varepsilon_{2,t+i}\right)\\
&= L\sigma_1^2 + \frac{b^2}{(1-\rho)^2}\left(L + \frac{\rho\left(1-\rho^L\right)\left(\rho^{L+1}-\rho-2\right)}{1-\rho^2}\right)\sigma_2^2.
\end{aligned} \tag{17}$$

□

We know from Lemma 1 that the variance of the lead time demand forecasting error is independent of time with the MMSE technique; hence, $\hat{\sigma}_t^L = \hat{\sigma}_{t'}^L$ $(t \neq t')$. Substituting Equation (15) into Equation (11) and according to $\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L$, we can achieve the following:

$$q_t^{MMSE} = -\frac{b\rho(1-\rho^L)}{1-\rho}(p_{t-1} - p_{t-2}) + d_{t-1}. \tag{18}$$

Theorem 1: *Suppose that the retailer uses the MMSE technique to forecast the lead time demand, the expression of the bullwhip effect on product orders is then the following*:

$$BWE_{order}^{MMSE} = \frac{Var\left(q_t^{MMSE}\right)}{Var(d_t)} = 1 + \frac{2b^2\rho(1-\rho^L)}{1-\rho}\left[\left(1-\rho^{L+1}\right)\right]\frac{\sigma_2^2}{(1-\rho^2)\sigma_1^2 + b^2\sigma_2^2}. \tag{19}$$

Proof: The variance of the product order quantity with the MMSE technique can be derived from Equation (18), as follows:

$$\begin{aligned}
Var\left(q_t^{MMSE}\right) &= \frac{b^2\rho^2\left(1-\rho^L\right)^2}{(1-\rho)^2}Var(p_{t-1} - p_{t-2})\\
&\quad + Var(d_{t-1}) - \frac{2b\rho\left(1-\rho^L\right)}{1-\rho}Cov(d_{t-1}, p_{t-1} - p_{t-2}),
\end{aligned} \tag{20}$$

where

$$Var(p_{t-1} - p_{t-2}) = 2Var(p_t) - 2Cov(p_{t-1}, p_{t-2}) = 2(1-\rho)Var(p_t) = 2\sigma_2^2/(1+\rho), \tag{21}$$

$$\begin{aligned}
Cov(d_{t-1}, p_{t-1} - p_{t-2}) &= Cov\left(a - bp_{t-1} + \varepsilon_{1,t-1}, p_{t-1} - p_{t-2}\right)\\
&= -bVar(p_t) + bCov(p_{t-1}, p_{t-2})\\
&= -b(1-\rho)Var(p_t) = -b\sigma_2^2/(1+\rho).
\end{aligned} \tag{22}$$

Substituting Equations (21) and (22) into Equation (20) and due to $Var(d_{t-1}) = Var(d_t) = \sigma_1^2 + b^2\sigma_2^2/(1-\rho^2)$, we obtain this theorem.

From the expression of the bullwhip effect on product orders in Theorem 1, we know that with the MMSE forecasting technique the bullwhip effect on product orders depends on the following five parameters: the price sensitivity coefficient b, the price correlation coefficient ρ, the lead time L and the variance of error terms σ_1^2 and σ_2^2. However, the market demand scale a has no effect on BWE_{order}^{MMSE}.

5.2 *The bullwhip effect on product orders using the MA forecasting technique*

MA is a forecasting technique that uses an average of actual observations from a specified number of prior periods. Using the MA technique, the demand forecast of period $t+i$ ($i = 0, 1, 2, \ldots$) made at the end of period $t-1$ can be expressed as the following:

$$\hat{d}_{t+i} = \sum\nolimits_{j=1}^{K} d_{t-j}/K, \quad i = 0, 1, 2, \ldots, \tag{23}$$

where K is the number of historical data used in the MA technique.

It should be noted that the forecasts for periods $t+i (i = 0, 1, 2, \ldots)$ made at time $t-1$ are equal: $\hat{d}_t = \hat{d}_{t+1} = \hat{d}_{t+2} = \cdots$. Therefore, a forecast of the total demand over L periods after period $t-1$ can be given as follows:

$$\hat{D}_t^L = \sum_{i=0}^{L-1} \hat{d}_{t+i} = L \times \left(\sum_{i=1}^{K} d_{t-i}/K \right). \tag{24}$$

Lemma 2: *When the MA technique is used to forecast the lead time demand, the variance of the forecasting error for the lead time demand remains constant over time and is given by the following*:

$$\left(\hat{\sigma}_t^L\right)^2 = \left(L + \frac{L^2}{K}\right)\sigma_d^2 + \frac{2b^2\rho}{1-\rho}\left(L + \frac{L^2}{K} - \frac{1-\rho^L}{1-\rho} - \frac{L}{K}\left(\frac{L}{K} + 1 - \rho^L\right)\frac{1-\rho^K}{1-\rho}\right)\sigma_p^2. \tag{25}$$

where $\sigma_d^2 = \sigma_1^2 + b^2\sigma_2^2/(1-\rho^2)$, $\sigma_p^2 = \sigma_2^2/(1-\rho^2)$.

Proof: The variance of the forecasting error for the lead time demand is the following:

$$\left(\hat{\sigma}_t^L\right)^2 = Var\left(D_t^L - \hat{D}_t^L\right) = Var\left(D_t^L\right) + Var\left(\hat{D}_t^L\right) - 2Cov\left(D_t^L, \hat{D}_t^L\right), \tag{26}$$

where

$$Var\left(D_t^L\right) = Var\left(\sum_{i=0}^{L-1} d_{t+i}\right) = \sum_{i=0}^{L-1} Var(d_{t+i}) + 2\sum_{i=0}^{L-2}\sum_{j=1}^{L-1-i} Cov\left(d_{t+i}, d_{t+i+j}\right)$$

$$= L\sigma_d^2 + 2b^2\sigma_p^2 \sum_{i=0}^{L-2}\sum_{j=1}^{L-1-i} \rho^j = L\sigma_d^2 + \left(2b^2\rho/(1-\rho)\right)\left(L - (1-\rho^L)/(1-\rho)\right)\sigma_p^2,$$

$$Var\left(\hat{D}_t^L\right) = (L/K)^2 Var\left(\sum_{i=1}^{K} d_{t-i}\right) = (L/K)^2\left[K\sigma_d^2 + \left(2b^2\rho/(1-\rho)\right)\left(K - \left(1-\rho^K\right)/(1-\rho)\right)\sigma_p^2\right],$$

$$Cov\left(D_t^L, \hat{D}_t^L\right) = Cov\left(\sum_{i=0}^{L-1} d_{t+i}, L\left(\sum_{i=1}^{K} d_{t-i}/K\right)\right) = (L/K)\sum_{i=0}^{L-1}\sum_{j=1}^{K} Cov\left(d_{t+i}, d_{t-j}\right)$$

$$= (L/K)b^2\sigma_p^2 \sum_{i=0}^{L-1}\sum_{j=1}^{K} \rho^{i+j} = (L/K)\left(b^2\rho\left(1-\rho^L\right)\left(1-\rho^K\right)/(1-\rho)^2\right)\sigma_p^2.$$

Substituting these three terms into Equation (26), we know that $(\hat{\sigma}_t^L)^2$ remains constant over time. □

We know from Lemma 2 that the variance of the forecasting error for lead time demand is also independent of time using the MA technique: $\hat{\sigma}_t^L = \hat{\sigma}_{t-1}^L$. Substituting Equation (24) into Equation (11), we obtain the following:

$$q_t^{MA} = (1 + L/K)d_{t-1} - (L/K)d_{t-K-1}. \tag{27}$$

Theorem 2: *Suppose that the retailer uses the MA technique to forecast the lead time demand, the expression of the bullwhip effect on product orders is then the following*:

$$BWE_{order}^{MA} = \frac{Var\left(q_t^{MA}\right)}{Var(d_t)} = 1 + \left(\frac{2L}{K} + \frac{2L^2}{K^2}\right)\left(1 - \frac{b^2\rho^K\sigma_2^2}{(1-\rho^2)\sigma_1^2 + b^2\sigma_2^2}\right). \tag{28}$$

Proof: The variance of the product order quantity using the MA technique can be derived from Equation (27) as follows:

$$\begin{aligned} Var\left(q_t^{MA}\right) &= (1 + L/K)^2 Var(d_{t-1}) + (L/K)^2 Var(d_{t-K-1}) \\ &\quad - 2L/K(1 + L/K)Cov(d_{t-1}, d_{t-K-1}) \\ &= \left(1 + 2L/K + 2L^2/K^2\right)Var(d_t) - \left(2L/K + 2L^2/K^2\right)Cov(d_{t-1}, d_{t-K-1}), \end{aligned} \tag{29}$$

where

$$\begin{aligned} Cov(d_{t-1}, d_{t-K-1}) &= Cov\left(a - bp_{t-1} + \varepsilon_{1,t-1}, a - bp_{t-K-1} + \varepsilon_{1,t-K-1}\right) \\ &= b^2 Cov(p_{t-1}, p_{t-K-1}) = b^2\rho^K\sigma_2^2/\left(1-\rho^2\right). \end{aligned} \tag{30}$$

Substituting Equation (30) into Equation (29) and due to $Var(d_t) = \sigma_1^2 + b^2\sigma_2^2/(1-\rho^2)$, we can prove this theorem.

From the expression of the bullwhip effect on product orders in Theorem 2, we can work out that with the MA forecasting technique the BWE_{order}^{MA} depends on the following six parameters: the price sensitivity coefficient b, the price correlation coefficient ρ, the lead time L, the number of observations K used in the MA technique and the variance of error terms σ_1^2 and σ_2^2. The market demand scale a also has no effect on BWE_{order}^{MA}.

5.3 *The bullwhip effect on product orders using the ES forecasting technique*

ES is a forecasting technique that uses a weighted moving average of past data as the basis for a forecast. Using the ES technique, the forecasting model can be expressed as the following:

$$\hat{d}_{t+i} = \alpha d_{t-1} + (1-\alpha)\hat{d}_{t-1}, \quad i = 0, 1, 2, \ldots, \tag{31}$$

where α is the smoothing coefficient ($0 < \alpha < 1$).

Also note that the forecasts for periods $t+i$ ($i = 0, 1, 2, \ldots$) made at time $t-1$ are equal: $\hat{d}_t = \hat{d}_{t+1} = \hat{d}_{t+2} = \cdots$. In particular, we have $\hat{d}_t = \alpha d_{t-1} + (1-\alpha)\hat{d}_{t-1}$, which considers the variances of both sides of the equation. From this, we obtain $Var(\hat{d}_t) = (\alpha/(2-\alpha))Var(d_t) + (2(1-\alpha)/(2-\alpha))Cov(d_{t-1}, \hat{d}_{t-1})^5$ and the estimate of the mean lead time demand after period $t-1$:

$$\hat{D}_t^L = \sum_{i=0}^{L-1}\hat{d}_{t+i} = L\hat{d}_t. \tag{32}$$

Lemma 3: *When the ES technique is used to forecast the lead time demand, the variance of the forecasting error for the lead time demand remains constant over time and is given by the following:*

$$\left(\hat{\sigma}_t^L\right)^2 = \left(L + \frac{\alpha}{2-\alpha}L^2\right)\sigma_d^2 + 2b^2\rho\left(\frac{1}{1-\rho}\left(L - \frac{1-\rho^L}{1-\rho}\right) + \frac{\alpha L}{1-(1-\alpha)\rho}\left(\frac{1-\alpha}{2-\alpha}L - \frac{1-\rho^L}{1-\rho}\right)\right)\sigma_p^2. \tag{33}$$

where $\sigma_d^2 = \sigma_1^2 + b^2\sigma_2^2/(1-\rho^2)$, $\sigma_p^2 = \sigma_2^2/(1-\rho^2)$.

Proof: The variance of the forecasting error for the lead time demand is the following:

$$\left(\hat{\sigma}_t^L\right)^2 = Var\left(D_t^L\right) + Var\left(\hat{D}_t^L\right) - 2Cov\left(D_t^L, \hat{D}_t^L\right), \tag{34}$$

where

$$Var\left(D_t^L\right) = L\sigma_d^2 + \left(2b^2\rho/(1-\rho)\right)\left(L - \left(1-\rho^L\right)/(1-\rho)\right)\sigma_p^2, \tag{35}$$

$$\begin{aligned} Var\left(\hat{D}_t^L\right) &= L^2 Var(\hat{d}_t) = L^2\left[(\alpha/(2-\alpha))Var(d_t) + (2(1-\alpha)/(2-\alpha))Cov\left(d_{t-1}, \hat{d}_{t-1}\right)\right] \\ &= L^2\left[(\alpha/(2-\alpha))\sigma_d^2 + (2(1-\alpha)/(2-\alpha))\left(\alpha b^2\rho/(1-(1-\alpha)\rho)\right)\sigma_p^2\right], \end{aligned} \tag{36}$$

$$\begin{aligned} Cov\left(D_t^L, \hat{D}_t^L\right) &= Cov\left(\sum_{i=0}^{L-1} d_{t+i}, L\hat{d}_t\right) = L\sum_{i=0}^{L-1} Cov\left(d_{t+i}, \hat{d}_t\right) \\ &= L\alpha b^2 \sum_{i=0}^{L-1}\sum_{j=1}^{\infty}(1-\alpha)^{j-1}\rho^{i+j}\sigma_p^2 = \frac{L\alpha b^2\rho(1-\rho^L)}{(1-\rho)(1-(1-\alpha)\rho)}\sigma_p^2, \end{aligned} \tag{37}$$

where

$$\begin{aligned} Cov\left(d_{t-1}, \hat{d}_{t-1}\right) &= Cov\left(d_{t-1}, \alpha\sum_{j=1}^{\infty}(1-\alpha)^{j-1}d_{t-j-1}\right) = \alpha\sum_{j=1}^{\infty}(1-\alpha)^{j-1}Cov\left(d_{t-1}, d_{t-j-1}\right) \\ &= \alpha b^2\sum_{j=1}^{\infty}(1-\alpha)^{j-1}\rho^j\sigma_p^2 = \left[\alpha b^2\rho/(1-(1-\alpha)\rho)\right]\sigma_p^2, \\ Cov\left(d_{t+i}, \hat{d}_t\right) &= Cov\left(d_{t+i}, \alpha\sum_{j=1}^{\infty}(1-\alpha)^{j-1}d_{t-j}\right) = \alpha\sum_{j=1}^{\infty}(1-\alpha)^{j-1}Cov\left(d_{t+i}, d_{t-j}\right) \\ &= \alpha b^2\sum_{j=1}^{\infty}(1-\alpha)^{j-1}\rho^{i+j}\sigma_p^2. \end{aligned}$$

Substituting Equations (35), (36) and (37) into Equation (34), we find that $(\hat{\sigma}_t^L)^2$ remains constant over time. □

We know from Lemma 3 that $\hat{\sigma}_t^L = \hat{\sigma}_{t-1}^L$ using the ES technique as well. Substituting Equation (32) into Equation (11), we obtain the following:

$$q_t^{ES} = L\left(\hat{d}_t - \hat{d}_{t-1}\right) + d_{t-1} = L\left[\left(\alpha d_{t-1} + (1-\alpha)\hat{d}_{t-1}\right) - \hat{d}_{t-1}\right] + d_{t-1} = (1+\alpha L)d_{t-1} - \alpha L\hat{d}_{t-1}. \tag{38}$$

Theorem 3: *Suppose that the retailer uses the ES technique to forecast the lead time demand, the expression of the bullwhip effect on product orders is then the following*:

$$BWE_{order}^{ES} = \frac{Var(q_t^{ES})}{Var(d_t)} = 1 + 2\alpha L\left(1 + \frac{\alpha L}{2-\alpha}\right)\left(1 - \frac{\alpha b^2\rho}{1-(1-\alpha)\rho} \times \frac{\sigma_2^2}{(1-\rho^2)\sigma_1^2 + b^2\sigma_2^2}\right). \tag{39}$$

Proof: The variance of the product order quantity using the ES technique can be derived from Equation (38) in the following manner:

$$\begin{aligned} Var\left(q_t^{ES}\right) &= (1+\alpha L)^2 Var(d_t) + \alpha^2L^2 Var\left(\hat{d}_t\right) - 2\alpha L(1+\alpha L)Cov\left(d_{t-1}, \hat{d}_{t-1}\right) \\ &= \left(1 + 2\alpha L + 2\alpha^2L^2/(2-\alpha)\right)Var(d_t) - 2\alpha L(1+\alpha L/(2-\alpha))Cov\left(d_{t-1}, \hat{d}_{t-1}\right). \end{aligned} \tag{40}$$

Because $Cov(d_{t-1}, \hat{d}_{t-1}) = [\alpha b^2\rho/(1-(1-\alpha)\rho)]\sigma_p^2 = \alpha b^2\rho\sigma_2^2/[(1-\rho^2)(1-(1-\alpha)\rho)]$ and because $Var(d_t) = \sigma_1^2 + b^2\sigma_2^2/(1-\rho^2)$, we can prove this theorem.

From Theorem 3, we know that the BWE_{order}^{ES} has no relation to the market demand scale a, but, rather, it depends on the price sensitivity coefficient b, the price correlation coefficient ρ, the lead time L, the smoothing coefficient α and the variance of error terms σ_1^2 and σ_2^2.

5.4 *Comparison of the bullwhip effect on product orders using the three forecasting techniques*

We have derived the analytical expressions of the bullwhip effect on product orders with the MMSE, MA and ES techniques. For the three forecasting techniques, the increase in product order variability does not depend on the value of the market demand scale a, but, rather, it depends on the value of the price sensitivity coefficient b, the price correlation parameter ρ, the lead time L, the demand information used to construct the forecast (K or α) and the variances σ_1^2 and σ_2^2.

One of the objectives of this paper is to develop insights into analysing the impact of different forecasting techniques on the bullwhip effect on product orders by considering certain distinct characteristics of the product. To understand this point, consider the following example. Consider the two two-level supply chains that were described in Section 3, where each supply chain distributes a single product. The market demand scale, the price sensitivity coefficient, the price correlation parameter, the lead time and the variance of error terms may be different for these two kinds of product; consider how these different characteristics of each product influence the bullwhip effect on product orders. As stated above, the market demand scale does not influence the bullwhip effect on product orders. To facilitate the analysis, we assume that the lead times and the error terms are identical for different product characteristics.[6] If we let $\sigma_1^2 = \sigma_2^2$, Figure 1 describes the influence of the parameters b and ρ on BWE_{order}^{MMSE}, BWE_{order}^{MA} and BWE_{order}^{ES} when $L=2$ and $K=4$ and 5.

The following properties can be obtained from Figure 1 and from the expressions of BWE_{order}^{MMSE}, BWE_{order}^{MA} and BWE_{order}^{ES}:

(1) Suppose that the retailer uses the MMSE technique to forecast the lead time demand, then $\lim_{\rho\to 1^-} BWE_{order}^{MMSE} = \lim_{\rho\to -1^+} BWE_{order}^{MMSE} = BWE_{order}^{MMSE}|_{\rho=0} = 1$. The BWE_{order}^{MMSE} increases as b increases in the case of $\rho > 0$, while it decreases as b increases in the case of $\rho < 0$. Note that the bullwhip effect on product orders does not exist in the case of $\rho < 0$.

(2) When the smoothing parameter α is selected as $\alpha = 2/(K+1)$ to equalise the average age of data used in the MA and ES techniques (Chen *et al.* 2000b, Zhang 2004), the increase in product order variability with MA technique is smaller than that with the ES technique, which coincides with the conclusion made by Chen *et al.* (2000b) and Zhang (2004). In the case of $\rho > 0$, the bullwhip effect on product orders for the two forecasting techniques should tend to be consistent in the case of $\rho \to 1^-$.

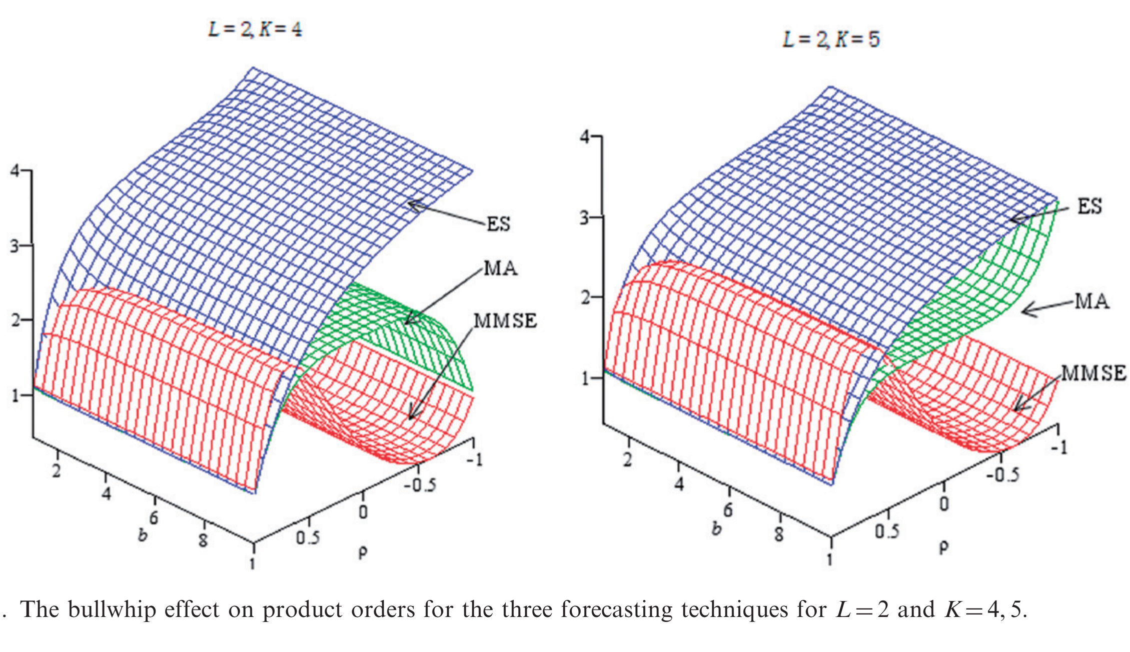

Figure 1. The bullwhip effect on product orders for the three forecasting techniques for $L=2$ and $K=4, 5$.

The manufacturer wants the forecasting technique that the retailer has chosen to minimise the product order variance amplification. Note that the increase in product order variability with the MA technique is always smaller than that with the ES technique; therefore, the retailer should choose the forecasting technique MMSE or MA. Proposition 1 outlines the conditions under which the retailer should choose the MMSE technique.

Proposition 1: *The retailer should choose the MMSE technique to minimise the product order variance amplification, if the following condition holds*:

$$\frac{b^2\rho(1-\rho^L)(1-\rho^{L+1})}{(1-\rho)((1-\rho^2)+b^2(1-\rho^K))} \leq \frac{L}{K}+\frac{L^2}{K^2}. \tag{41}$$

Proposition 1 can be obtained by letting $BWE_{order}^{MMSE} \leq BWE_{order}^{MA}$, where BWE_{order}^{MMSE} is given by Equation (19) and BWE_{order}^{MA} is given by Equation (28). If the price correlation coefficient $\rho < 0$, we see that the condition in Proposition 1 always holds. Hence, in this case, the retailer should choose the MMSE technique. Otherwise, if the price correlation coefficient $\rho > 0$, we see that the retailer should choose the MA technique if the condition in Proposition 1 does not hold.[7]

6. Analysis of the bullwhip effect on inventory at the retailer

We use a new definition of an inventory oscillations measure based on the definition of the bullwhip effect on product orders. We define it as follows:

$$BWE_{inventory} = Var(I_t)/Var(d_t). \tag{42}$$

The bullwhip effect on inventory is computed as the ratio of the inventory variance of the retailer to the demand variance of the customer. We introduce the definition of the bullwhip effect on an inventory similar to Gilbert (2005), so that the bullwhip effect on inventory can be applied to quantify fluctuations in actual inventory of the retailer, which is shown in Equation (42); see, e.g., Disney and Towill (2003) and Coppini *et al.* (2010). An increased inventory variance results in higher levels of safety stock and makes larger inventory costs unavoidable for the retailer (Hoberg *et al.* 2007); therefore, the introduction of the bullwhip effect on inventory yields some insights into the consequences that the inventory oscillations have on inventory management costs. The retailer wants the chosen forecasting technique to restrain the net inventory variance amplification; thus, in what follows, we analyse the bullwhip effect on inventory when the MMSE, MA and ES techniques are used to forecast the lead time demand.

Note that the net inventory variance and variance of the lead time demand forecasting error are identical. Applying Lemmas 1, 2 and 3, we obtain the following theorems.

Theorem 4: *Suppose that the retailer uses the MMSE technique to forecast the lead time demand, the expression of the bullwhip effect on inventory can then be given as follows*:

$$BWE_{inventory}^{MMSE} = \frac{Var(I_t^{MMSE})}{Var(d_t)} = \frac{(\hat{\sigma}_t^L)^2}{\sigma_d^2} = \frac{L\sigma_1^2 + \frac{b^2}{(1-\rho)^2}\left(L + \frac{\rho(1-\rho^L)(\rho^{L+1}-\rho-2)}{1-\rho^2}\right)\sigma_2^2}{\sigma_1^2 + \frac{b^2}{1-\rho^2}\sigma_2^2}. \tag{43}$$

Theorem 5: *Suppose that the retailer uses the MA technique to forecast the lead time demand, the expression of the bullwhip effect on inventory can then be given as follows*:

$$\begin{aligned} BWE_{inventory}^{MA} &= \frac{Var(I_t^{MA})}{Var(d_t)} = \frac{(\hat{\sigma}_t^L)}{\sigma_d^2} \\ &= L + \frac{L^2}{K} + \frac{2b^2\rho}{1-\rho}\left(L + \frac{L^2}{K} - \frac{1-\rho^L}{1-\rho} - \frac{L}{K}\left(\frac{L}{K} + 1 - \rho^L\right)\frac{1-\rho^K}{1-\rho}\right)\frac{\sigma_2^2}{(1-\rho^2)\sigma_1^2 + b^2\sigma_2^2}. \end{aligned} \tag{44}$$

Theorem 6: *Suppose that the retailer uses the ES technique to forecast the lead time demand, the expression of the bullwhip effect on inventory can then be given as follows*:

$$BME^{ES}_{inventory} = \frac{Var\left(I^{ES}_t\right)}{Var(d_t)} = \frac{\left(\hat{\sigma}^L_t\right)}{\sigma^2_d} = L + \frac{\alpha}{2-\alpha}L^2 + 2b^2\rho\left(\frac{1}{1-\rho}\left(L - \frac{1-\rho^L}{1-\rho}\right) + \frac{\alpha L}{1-(1-\alpha)\rho}\left(\frac{1-\alpha}{2-\alpha}L - \frac{1-\rho^L}{1-\rho}\right)\right)\frac{\sigma^2_2}{(1-\rho^2)\sigma^2_1 + b^2\sigma^2_2}. \tag{45}$$

Notice that, according to the above theorems, the market demand scale a also has no effect on the bullwhip effect on inventory for the three forecasting techniques.

6.1 *Comparison of the bullwhip effect on inventory using the three forecasting techniques*

In what follows, we compare $BME^{MMSE}_{inventory}$, $BME^{MA}_{inventory}$ and $BME^{ES}_{inventory}$ when $L=2$ and 3, $K=4$ and 5 ($\alpha = 2/(K+1)$ so that $\alpha = 0.4$ and 0.33, respectively).[8] We assume that $\sigma^2_1 = \sigma^2_2$. According to the expressions of the bullwhip effect on inventory in Theorems 4, 5 and 6, Figure 2 depicts these three different measures as a function of b and ρ.

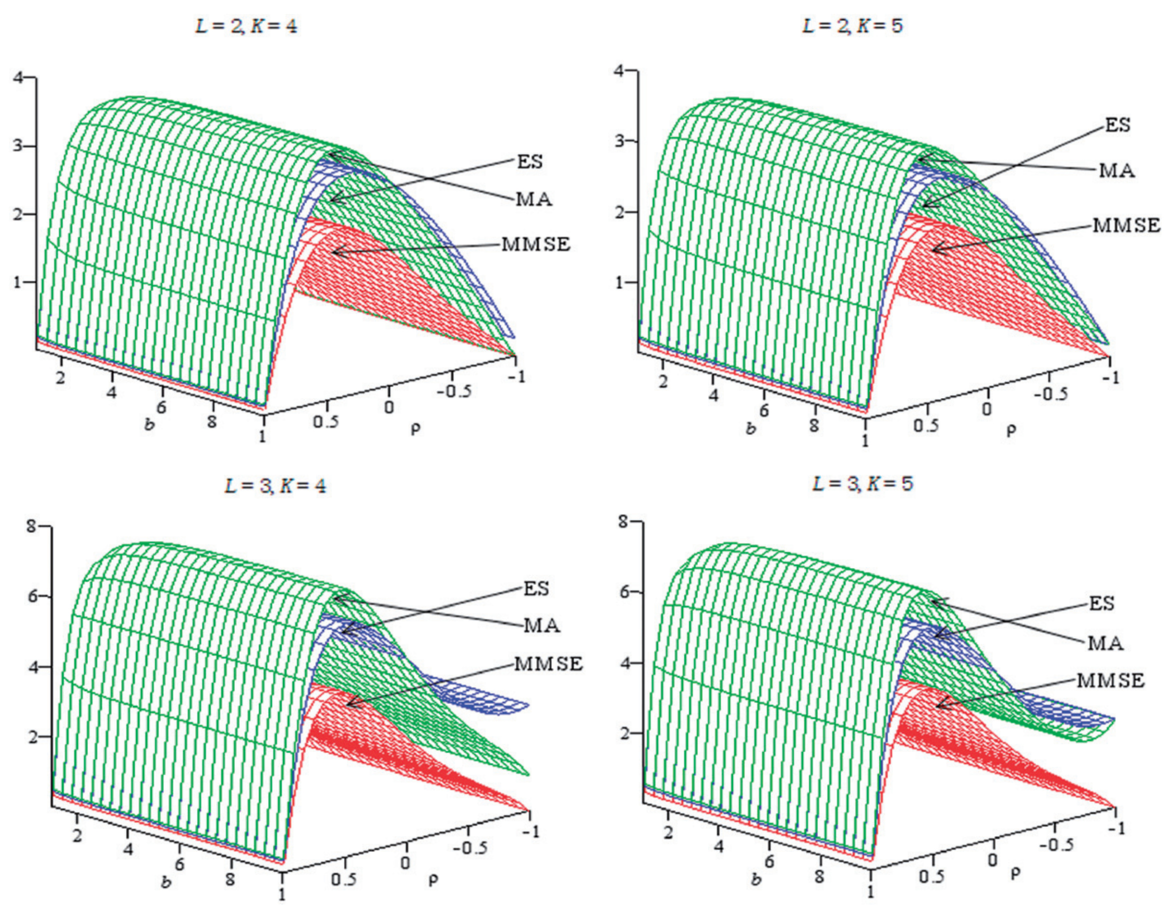

Figure 2. The bullwhip effect on inventory for the three forecasting techniques for $L=2,3$ and $K=4,5$.

We can obtain the following properties from Figure 2 and from the expressions of the $BME^{MMSE}_{inventory}$, $BME^{MA}_{inventory}$ and $BME^{ES}_{inventory}$:

(1) The bullwhip effect on inventory with the MMSE technique is smaller than the effect with the MA or ES techniques.
(2) Comparing the bullwhip effect on inventory using the MA technique with that using the ES technique: in the case of $\rho > 0$, $BME^{MA}_{inventory} > BME^{ES}_{inventory}$; in the case of $\rho = 0$, $BME^{MA}_{inventory} = BME^{ES}_{inventory}$; in the case of $\rho < 0$, $BME^{MA}_{inventory} < BME^{ES}_{inventory}$.

Because the MMSE technique minimises the retailer's demand forecasting error, it leads to the lowest bullwhip effect on inventory. Therefore, from the perspective of the retailer, they should choose the MMSE technique to forecast the lead time demand.

7. Balancing the bullwhip effect on product orders and inventory

In this section, we propose a measure of the bullwhip effect based on the weighted sum of the two variance amplifications and derive the conditions for choosing among the three forecasting techniques the one that best minimises the bullwhip effect.

7.1 *Performance measure for the bullwhip effect on product orders and inventory*

An obvious question to ask now is which forecasting technique should the retailer choose to minimise the sum of the bullwhip effect on product orders and inventory under different combinations of the parameters (b, ρ and L). We use Equation (46) to evaluate the whole performance measure (termed BWE) for the bullwhip effect on product orders (termed BWE_{order}) and the bullwhip effect on inventory ($BWE_{inventory}$) under different weightings (θ).

$$BWE = \theta \cdot BWE_{order} + (1 - \theta) \cdot BWE_{inventory}, \quad 0 \leq \theta \leq 1, \tag{46}$$

where

$$BWE_{order} = Var(q_t)/Var(d_t), \tag{47}$$

$$BWE_{inventory} = Var(I_t)/Var(d_t). \tag{48}$$

Previous research on the bullwhip effect on product orders relies on the perspective of the supply chain upstream because the bullwhip effect on product orders mainly contributes to upstream costs (Lee *et al*. 2000, Kim and Ryan 2003, Zhang 2004). However, this paper introduces Equation (46) as a new performance measure of the supply chain. Therefore, we can analyse the retailer's forecasting techniques from the perspective of the entire supply chain, i.e. both from upstream and downstream perspectives of the supply chain. When the value of θ is comparatively greater, the supply chain pays more attention to its effect on the upstream manufacturer; otherwise, the supply chain pays more attention to its effect on the downstream retailer. For instance, if the manufacturer in the upstream business dominates in the supply chain, the supply chain will focus its efforts on reducing the product order variance amplification. In other words, the retailer should choose a forecasting technique to minimise the sum of the bullwhip effect on product orders, and the bullwhip effect on inventory with a comparatively greater θ value. However, if the retailer dominates in the supply chain, the supply chain will focus its efforts on reducing the bullwhip effect on inventory with a comparatively smaller θ value.

According to Equation (46), we can give the performance measure of the sum of the two amplifications for the three forecasting techniques. For example, we can display the performance measure with the MMSE technique: $BWE^{MMSE} = \theta \cdot BWE^{MMSE}_{order} + (1 - \theta) \cdot BWE^{MMSE}_{inventory}$, where $BWE^{MMSE}_{inventory}$ is the expression of the bullwhip effect on product orders in Theorem 1, and $BWE^{MMSE}_{inventory}$ is the expression of the bullwhip effect on inventory in Theorem 4.

To obtain the conditions under which the retailer should choose a forecasting technique to minimise the sum of the two oscillations, the retailer should make $\alpha = 2/(K + 1)$ so that the average age of data used in the MA and ES techniques are equal (Chen *et al*. 2000b, Zhang 2004) and make $\sigma_1^2 = \sigma_2^2$ to facilitate the analysis. Using Equation (46), we can display the performance measure with the MMSE, MA and ES techniques, i.e. BWE^{MMSE}, BWE^{MA} and BWE^{ES}. Letting $BWE^{MMSE} \leq BWE^{MA}$ and $BWE^{MMSE} \leq BWE^{ES}$, with some algebra we can then obtain the two inequalities shown in Proposition 2.

Proposition 2: *The retailer should choose the MMSE technique to minimise the sum of the bullwhip effect on product orders and inventory under different weightings of θ, if the following conditions hold*:

$$2\theta \cdot f_1(b, \rho, L, K) + (1-\theta) \cdot f_2(b, \rho, L, K) \le 0, \tag{49}$$

$$2\theta \cdot f_3(b, \rho, L, K) + (1-\theta) \cdot f_4(b, \rho, L, K) \le 0. \tag{50}$$

Similarly, *Propositions* 3 *and* 4 *show the conditions in which the retailer should choose the MA or ES technique.*

Proposition 3: *The retailer should choose the MA technique to minimise the sum of the bullwhip effect on product orders and inventory under different weightings of θ, if the following conditions hold*:

$$2\theta \cdot f_1(b, \rho, L, K) + (1-\theta) \cdot f_2(b, \rho, L, K) \ge 0, \tag{51}$$

$$2\theta \cdot f_5(b, \rho, L, K) + (1-\theta) \cdot f_6(b, \rho, L, K) \le 0. \tag{52}$$

Proposition 4: *The retailer should choose the ES technique to minimise the sum of the bullwhip effect on product orders and inventory under different weightings of θ, if the following conditions hold*:

$$2\theta \cdot f_3(b, \rho, L, K) + (1-\theta) \cdot f_4(b, \rho, L, K) \ge 0, \tag{53}$$

$$2\theta \cdot f_5(b, \rho, L, K) + (1-\theta) \cdot f_6(b, \rho, L, K) \ge 0, \tag{54}$$

where

$$f_1(b,\rho,L,K) = \frac{b^2\rho(1-\rho^L)(1-\rho^{L+1})}{(1-\rho)(1-\rho^2+b^2)} - \left(1 - \frac{b^2\rho^K}{1-\rho^2+b^2}\right)\left(\frac{L}{K}+\frac{L^2}{K^2}\right),$$

$$f_2(b,\rho,L,K) = \frac{\frac{b^2(1+\rho)}{1-\rho}\left(\frac{\rho(1-\rho^L)(\rho^{L+1}-\rho-2)}{1-\rho^2}+L\right) - \frac{2b^2\rho}{1-\rho}\left(L+\frac{L^2}{K}-\frac{1-\rho^L}{1-\rho}-\frac{1-\rho^K}{1-\rho}\frac{L}{K}\left(\frac{L}{K}+1-\rho^L\right)\right) - b^2L}{1-\rho^2+b^2} - \frac{L^2}{K},$$

$$f_3(b,\rho,L,K) = \frac{b^2\rho(1-\rho^L)(1-\rho^{L+1})}{(1-\rho)(1-\rho^2+b^2)} - 2\left(1+\frac{2b^2\rho}{(\rho(K-1)-K-1)(1-\rho^2+b^2)}\right)\frac{L}{K+1}\left(1+\frac{L}{K}\right),$$

$$f_4(b,\rho,L,K) = \frac{\frac{b^2(1+\rho)}{1-\rho}\left(\frac{\rho(1-\rho^L)(\rho^{L+1}-\rho-2)}{1-\rho^2}+L\right) - 2b^2\rho\left(\frac{L-\frac{1-\rho^L}{1-\rho}}{1-\rho}+\frac{2L}{\rho(K-1)-K-1}\left(\frac{1-\rho^L}{1-\rho}-\frac{(K-1)L}{2K}\right)\right) - b^2L}{1-\rho^2+b^2} - \frac{L^2}{K},$$

$$f_5(b,\rho,L,K) = \left(1-\frac{b^2\rho^K}{1-\rho^2+b^2}\right)\left(\frac{L}{K}+\frac{L^2}{K^2}\right) - 2\left(1+\frac{2b^2\rho}{(\rho(K-1)-K-1)(1-\rho^2+b^2)}\right)\frac{L}{K+1}\left(1+\frac{L}{K}\right),$$

$$f_6(b,\rho,L,K) = \frac{\frac{2b^2\rho}{1-\rho}\left(L+\frac{L^2}{K}-\frac{1-\rho^L}{1-\rho}-\frac{1-\rho^K}{1-\rho}\frac{L}{K}\left(\frac{L}{K}+1-\rho^L\right)\right) - 2b^2\rho\left(\frac{L-\frac{1-\rho^L}{1-\rho}}{1-\rho}+\frac{2L}{\rho(K-1)-K-1}\left(\frac{1-\rho^L}{1-\rho}-\frac{(K-1)L}{2K}\right)\right)}{1-\rho^2+b^2}.$$

Especially in the case of $\rho < 0$, whether we consider the perspective that the manufacturer wishes to minimise the bullwhip effect on product orders or the perspective that the retailer wishes to minimise the bullwhip effect on inventory, the MMSE technique restrains the two oscillations contemporaneously. We have already shown the results at the end of Section 5 and at the end of Section 6, respectively. However, recall that we have already shown that our demand model in Equation (1) and price dynamics model in Equation (2) can be reduced to an autoregressive demand process, i.e. $d_t = \mu' + \lambda d_{t-1} + \varepsilon_t$, where $\mu' = a(1-\rho) - b\mu$, $\lambda = \rho$, $\varepsilon_t = \varepsilon_{1,t} - \rho\varepsilon_{1,t-1} - b\varepsilon_{2,t}$. As reported by Lee *et al*. (2000), it is common to have a positive demand correlation coefficient λ in a high-tech industry or for the sales pattern of most products. They found that λ varies from 0.26 to 0.89 for 150 stock keeping units (SKUs). Because $\lambda = \rho$, we can also deduce that the price correlation coefficient $\rho > 0$ is common. When the coefficient ρ is positive, the process is reflected by a wandering or meandering sequence of observations. We shall restrict our attention using numerical analysis to the case where $\rho > 0$ and explain the conditions shown in Propositions 2, 3 and 4.

7.2 *Numerical analysis*

To illustrate the conditions shown in Propositions 2, 3 and 4 in the case of $\rho > 0$, the following numerical example is studied.

Similar to our analysis in Sections 5 and 6, let $\alpha = 2/(K+1)$ and $\sigma_1^2 = \sigma_2^2$ when $K=2$, 4, 5 and 7 ($\alpha = 0.67$, 0.4, 0.33 and 0.25, respectively). The 'areas' (different combinations of the price sensitivity coefficient b and the price correlation coefficient ρ) for choosing the best forecasting technique are shown in Figure 3 for $\theta = 0.2$, 0.5 and 0.8 and for $L = 1$, 2, 3 and 4, respectively. Figure 3(a) shows the case when the supply chain pays less attention to the bullwhip effect on product orders for $\theta = 0.2$, i.e. pays less attention to its effect on the manufacturer. Figure 3(b) shows the case when the supply chain pays equal attention to the bullwhip effect on product orders and inventory for $\theta = 0.5$. Figure 3(c) shows the case when the supply chain pays more attention to the bullwhip effect on product orders for $\theta = 0.8$. It can be observed from the figures that sometimes the retailer should choose the MMSE technique (labelled 'MMSE' in each figure) under different combinations of b and ρ; in this case, the sum of the bullwhip effect on product orders and inventory is minimised with the MMSE technique. In other cases, the retailer may choose the MA or ES technique (labelled 'MA' or 'ES' in each figure). The *solid* line is the borderline between the area for choosing the MMSE technique and the area for choosing the MA technique. The *broken* line is the

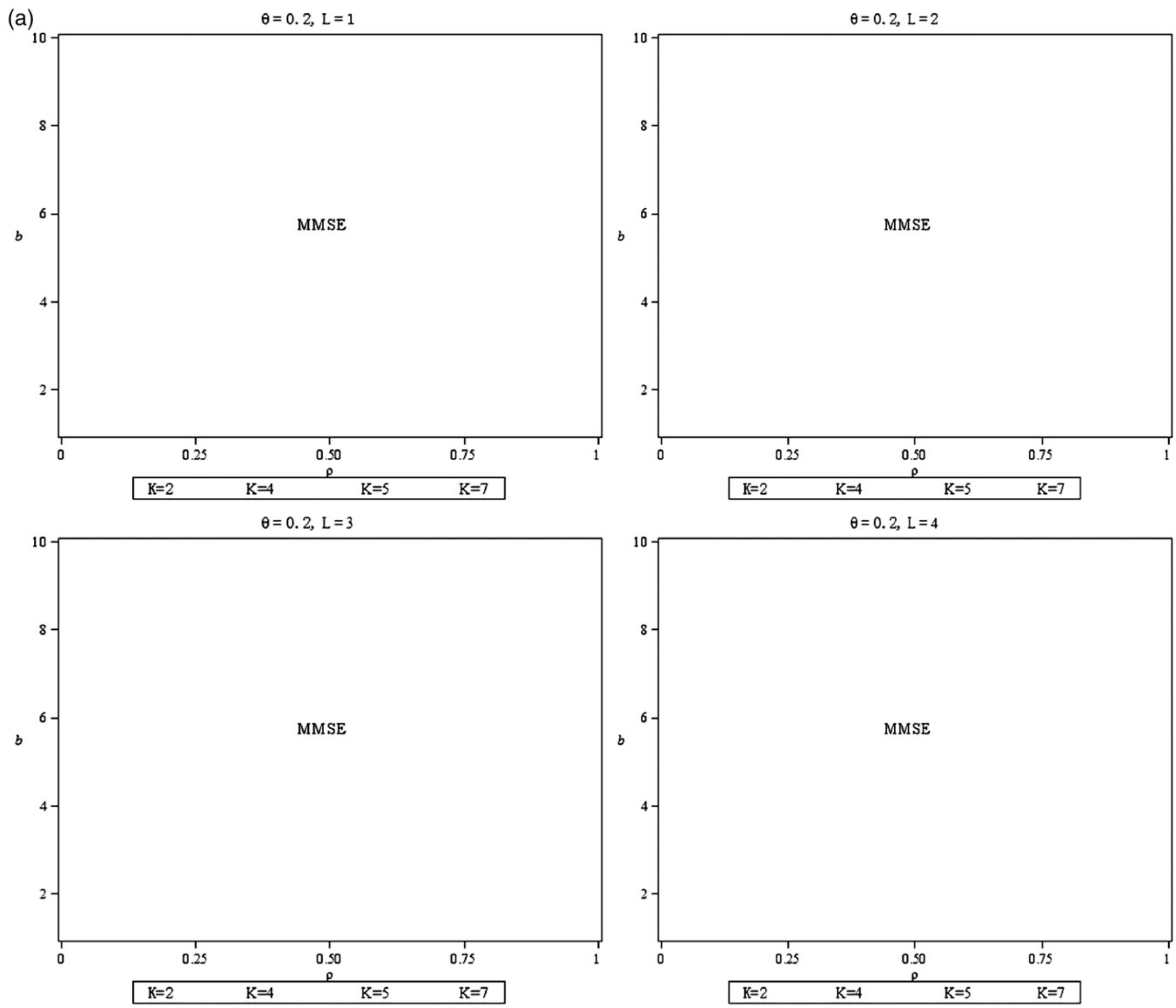

Figure 3. (a) The area for choosing the best forecasting technique for $\theta = 0.2$ and for $L = 1$, 2, 3 and 4. (b) The area for choosing the best forecasting technique for $\theta = 0.5$ and for $L = 1$, 2, 3 and 4. (c) The area for choosing the best forecasting technique for $\theta = 0.8$ and for $L = 1$, 2, 3 and 4.

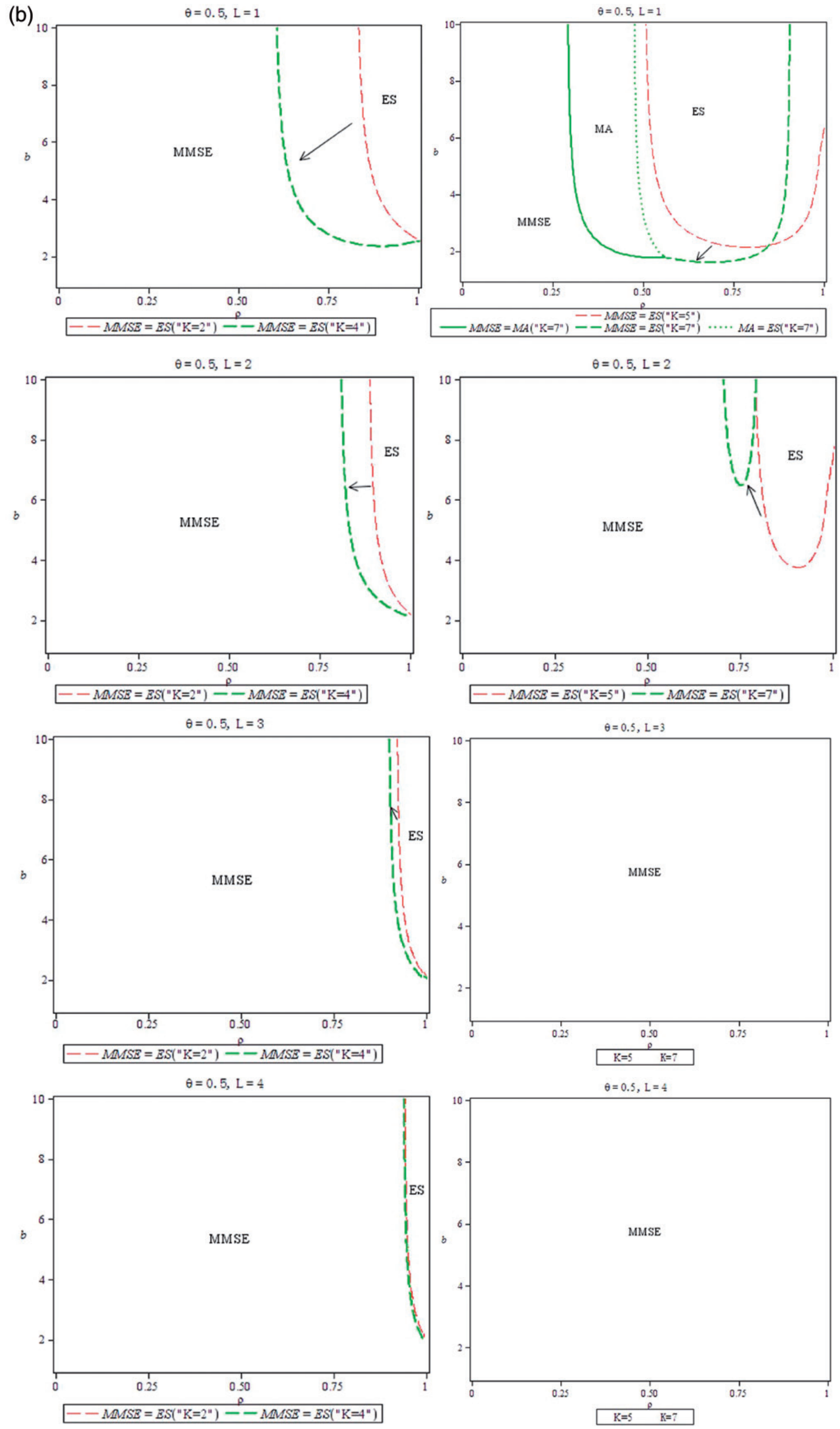

Figure 3. Continued.

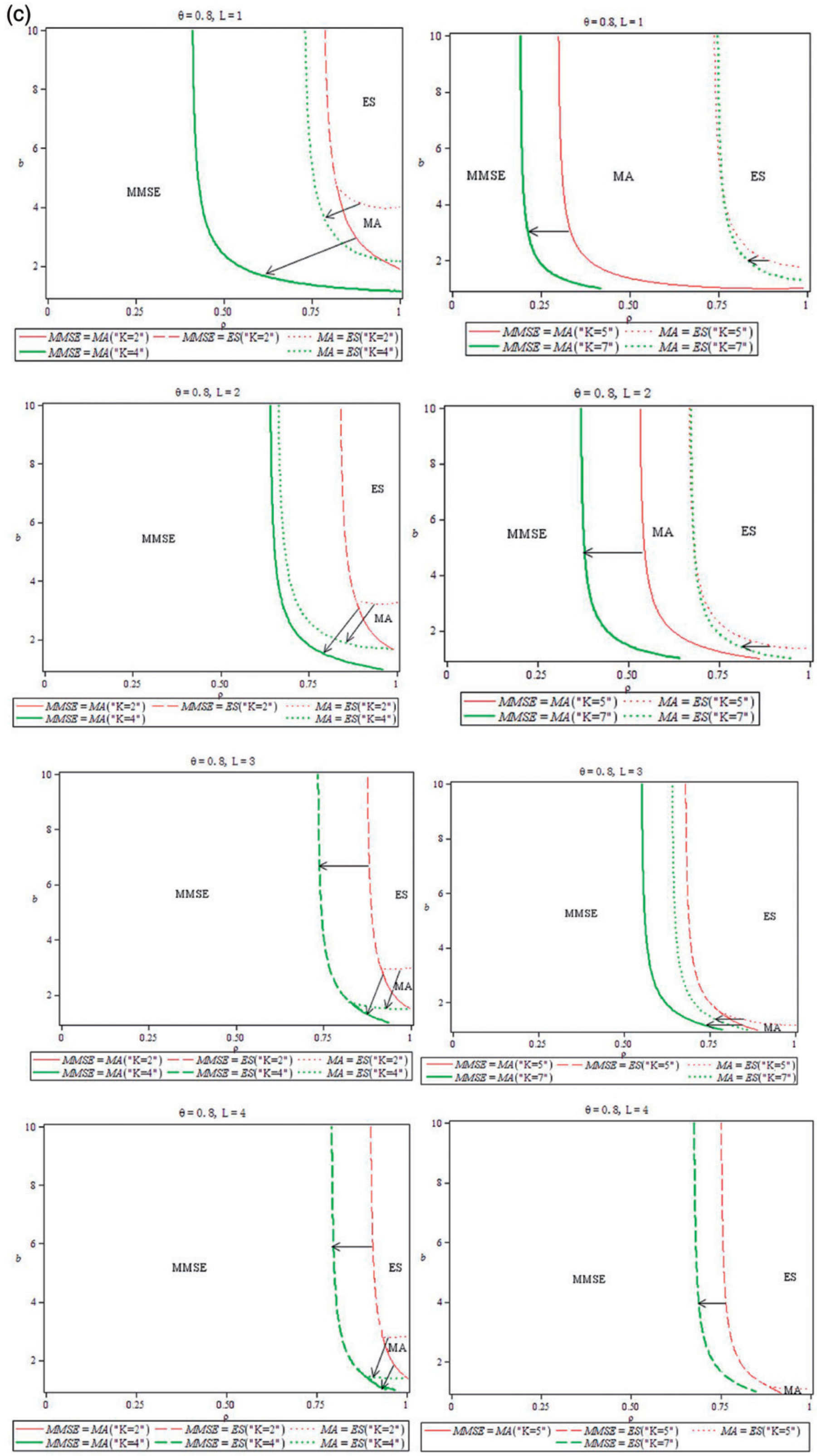

Figure 3. Continued.

borderline between the area for choosing the MMSE technique and the area for choosing the ES technique. The *dotted* line is the borderline between the area for choosing the MA technique and the area for choosing the ES technique. The thin (thick) line represents the situation in which the retailer relies less (more) on the history demand date to estimate the mean demand.

According to Figures 3(a), (b) and (c), we can see the following:

(1) Figure 3(a), in the case of $\theta = 0.2$ when $L = 1, 2, 3$ and 4. The retailer should choose the MMSE technique for a b value between 0 and 10 and for a ρ value between 0 and 1 because, when $\theta = 0.2$, the supply chain pays more attention to the bullwhip effect on inventory. Because the MMSE technique minimises the retailer's demand forecasting error, the net inventory variance is equal to the variance of the forecasting error. Therefore, the MMSE is the most appropriate technique for the retailer.

(2) Figure 3(b) in the case of $\theta = 0.5$. First, we consider the situation whereby the lead time $L = 1$ and the value of K is even. For $K = 2$, if the value of b varies from 2.5 to 10 and the value of ρ varies from 0.85 to 1, the retailer should choose the ES technique for this situation.[9] Otherwise, the retailer should choose the MMSE technique. For $K = 4$, the retailer can use historical demand information to conduct better forecasting. For this situation, the area for choosing the ES technique increases. In other words, the retailer should choose the ES technique if b ranges from 2.5 to 10 and ρ ranges from 0.6 to 1. In an analysis of the situation that $L = 1$ and the value of K is odd, for $K = 5$, the retailer should choose the ES technique if b ranges from 2.2 to 10 and ρ ranges from 0.52 to 1. For $K = 7$, the retailer should choose the MA technique if b ranges from 2 to 10 and ρ ranges from 0.3 to 0.5. Only when the value of b varies from 0 to 2, when the value of ρ varies from 0 to 0.3, or when the value of ρ varies from 0.9 to 1, should the retailer choose the MMSE technique.(b) Compare the situation when $L = 1$ ($L = 2$) with that when $L = 3$ ($L = 4$). As L increases from 1 to 3 (from 2 to 4) under the same K value, the area for choosing the MMSE increases. For products with a longer lead time, the MMSE technique is more applicable.

(3) Figure 3(c) in the case of $\theta = 0.8$. (a) This case is different from that of $\theta = 0.5$. The area for choosing the MMSE technique is decreased when the value of L is the same. If the supply chain pays more attention to its effect on the upstream manufacturer, the possibility of choosing the MA or ES technique is increased. For example, in the case of $\theta = 0.5$ for $L = 1$ and for $K = 2$ and 4, the retailer should not choose the MA technique. However, in the case of $\theta = 0.8$ for $L = 1$ and for $K = 2$ and 4, the retailer should choose the MA technique when ρ varies from 0.4 to 0.75 if more historical demand information can be used to forecast the lead time demand; see the situation that $K = 4$.(b) We will now compare the situation when $L = 1$ ($L = 2$) with that of $L = 3$ ($L = 4$) for $\theta = 0.8$. Under the same value of K, the increase of L (from 1 to 3 or from 2 to 4) does not have a detectable influence on the area for choosing the ES, but does decrease the area for choosing the MA and increase the area for choosing the MMSE. This also explains the situations whereby L is small and the retailer can use abundant historical demand information; see, e.g., the situation that $L = 1$ and $K = 4$, then the retailer should choose the MA technique if ρ ranges from 0.4 to 0.75, choose the MMSE technique if ρ ranges from 0 to 0.4, and choose the ES technique if ρ ranges from 0.75 to 1.

From the above analysis, several important managerial insights can be revealed when $\rho > 0$:

(1) If the supply chain pays less attention to its effect on the upstream manufacturer, or when the products' price sensitivity coefficient ranges from 0 to 2, i.e., the demand for this kind of product is not sensitive to the changing of price, then the retailer should choose the MMSE technique.

(2) For products with a price sensitivity coefficient ranging from 2 to 10, a price correlation coefficient ranging from 0.75 to 1, and if the supply chain pays more attention to its effect on the upstream manufacturer, then the retailer should choose the ES technique.

(3) When products with a price correlation coefficient ranging from 0.4 to 0.75 have a small lead time, abundant historical demand information, and if the supply chain pays more attention to its effect on the upstream manufacturer, the retailer should choose the MA technique.

8. Conclusions

The bullwhip effect on product orders mainly contributes to upstream costs, while the bullwhip effect on inventory increases downstream inventory costs. In this paper, we analysed the retailer's forecasting techniques (i.e. MMSE, MA and ES) based on the bullwhip effect on product orders and inventory from the perspective of the entire supply chain. We used the sum of the two variance amplifications under different weightings as the performance measure to decide which forecasting technique the retailer should choose for different product characteristics. Since the market demand scale has no influence on the bullwhip effect on product orders and inventory, it does not influence the retailer's choice. In addition, when the price correlation coefficient ρ is negative, the pricing process tends to exhibit period-to-period oscillation. Thus, in this case, the retailer should choose the MMSE technique to minimise the sum of the two variance amplifications. However, since the pricing process is often reflected by a wandering or meandering sequence of observations, the case of positive coefficient ρ is of considerable practical importance; therefore, we focus on studying this situation and showing that the retailer should choose the MMSE, MA or ES forecasting technique according to product characteristics. By only considering the bullwhip effect on product orders, the retailer should choose between the forecasting techniques MMSE and MA. More particularly, the retailer should choose the MA technique for products with a larger price sensitivity coefficient and a larger price correlation coefficient; otherwise, the retailer should choose the MMSE technique. On the other hand, by only considering the bullwhip effect on inventory, the retailer should choose the MMSE technique in all cases. However, we have shown that upstream businesses want the chosen forecasting technique to restrain the bullwhip effect on product orders, while downstream businesses want the chosen forecasting technique to restrain the bullwhip effect on inventory. Therefore, this study introduced the weighted sum of the two variance amplifications and obtained the conditions for choosing among the three techniques the one that best minimises the bullwhip effect:

(1) If the supply chain pays less attention to the bullwhip effect on product orders, or when the product price sensitivity coefficient is small, i.e. the demand for this product is not sensitive to any deviation in the price, the retailer should choose the MMSE technique.
(2) For products with a larger price sensitivity coefficient and a larger price correlation coefficient – if the supply chain pays more attention to the bullwhip effect on product orders – then the retailer should choose the ES technique.
(3) When products with a medium price correlation coefficient have a small lead time and abundant historical demand information – if the supply chain pays more attention to the bullwhip effect on product orders – the retailer should choose the MA technique.

The research presented here can lead in several directions that are likely to enhance our understanding of a retailer's forecasting techniques. First, this paper has shown that the bullwhip effect on product orders or on inventory mainly contributes to upstream or downstream costs. The methods for quantifying the impact of forecasting on the expected inventory costs in a supply chain still require further study. Second, this paper only considers the order-up-to inventory policy. Thus, other inventory policies must be analysed in further studies. Moreover, extending the two-level supply chain to multi-level chains and analysing the impact of information sharing is another direction for a follow-up paper.

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Notes

1. We use the terms 'inventory variance amplification', 'bullwhip effect on inventory' and 'inventory oscillations' depending on context.
2. Zhang and Burke (2011) also considered an AR (1) pricing process. However, they only focused on investigating compound causes of the bullwhip effect on product orders using the MMSE forecasting technique by considering an inventory system with multiple price-sensitive demand streams.
3. We use the stationary AR (1) pricing process to simplify our exposition. However, when the pricing process is nonstationary due to its increasing (or decreasing) trend, the mean price may vary over time. However, the process has a fixed slope γ that is known. After letting the mean price $E(p_t) = \mu_t = \omega + \gamma t$, we can know that the mean μ_t varies in a way that is known.

Therefore, we can use the same approach to analyse the case when the pricing process is nonstatinoary, say, $[p_t - \mu_t = \rho(p_{t-1} - \mu_{t-1}) + \varepsilon_{2,t}]$. It can be shown that the bullwhip effect on product orders and inventory remains unchanged. The demand model that with AR (1) demand process also used this approach to deal with the nonstationary situation, such as Sodhi and Tang (2011).

4. The parameters a and b can be estimated using previous observations $p_{t-1}, p_{t-2}, \ldots$ and $d_{t-1}, d_{t-2}, \ldots$. Thus, we can use the estimated regression equation to estimate $\hat{d}_{t+i}$ for a given $\hat{p}_{t+i}$ $(i = 0, 1, 2, \ldots)$.
5. Taking the variance of both sides of the equation $\hat{d}_t = \alpha d_{t-1} + (1-\alpha)\hat{d}_{t-1}$, we obtain $Var(\hat{d}_t) = \alpha^2 Var(d_{t-1}) + (1-\alpha)^2 Var(\hat{d}_{t-1}) + 2\alpha(1-\alpha)Cov(d_{t-1}, \hat{d}_{t-1})$. Recall that we have already assumed that the AR (1) pricing process is stationary. We notice from Equation (1) that the demand is a linear combination of the price and the error term. Thus, we can easily obtain $Var(d_{t-1}) = Var(d_t)$. Chen *et al.* (2000b) have shown that $Var(\hat{d}_{t-1})$ is the function of $Var(d_t)$, so that we can derive $Var(\hat{d}_{t-1}) = Var(\hat{d}_t)$. With some algebra, we can obtain this equation.
6. The assumption presented here can be extended to analyse different lead times and error terms; however, the analysis would become more complex. In fact, if we assume that the error terms are identical, we can get the same results when lead times are different. Since our intent is to obtain some basic managerial insights using simulation, we shall restrict our attention to the assumption that the lead times and error terms are identical.
7. The retailer should choose the MA technique, especially when b and ρ are increased. However, since our intent is to choose the one that best minimises both the variance amplification of the product orders and that of the inventory, we shall restrict the discussion here.
8. Let L or K be even or odd according to the expressions of the bullwhip effect on inventory for the three forecasting techniques, in particular for the MA and ES techniques.
9. Note, the 'areas' for choosing the best forecasting techniques are not accurate; there will be cases in which the retailer should choose the MMSE technique. See the situation when $b = 3$ and $\rho = 0.9$. However, since the sum of the two variance amplifications with the MMSE technique is close to that with the ES technique for these cases, we may say the retailer has chosen a near-optimal technique.

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